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Remarks on Mr. Winthrop's paper on the duplication of the Cube in part 1st of this volume. By GEORGE BARON, late Master of the Mathematical Academy at South Shields, in the County of Durham, in England.

MR. Winthrop has attempted the duplication of the Cube, in a paper inserted at page 25, part I. vol. 2, of the Memoirs of the American Academy of Arts and Sciences. This paper is entitled "Geometrical methods of finding any required series of mean proportionals between given extremes." It consists of four problems. In problem 1. is shewn the well known method of finding one mean proportional between two given extremes. Problem 2. is "To find two mean proportionals between two given extremes." On the truth of this problem the two remaining problems, together with Mr. Winthrop's duplication of the Cube, entirely depend. I shall therefore here, first describe Mr. Winthrop's method of finding two mean proportionals between two given extremes: Secondly, shew that the demonstration which he has attempted to give of that method is not true. And thirdly, I shall demonstrate that that method is universally false. And hence it will follow, that Mr. Winthrop's duplication of the Cube is universally false.

First then, I am to describe Mr. Winthrop's method of finding two mean proportionals between two given extremes. The substance of it is as follows.

Let

Plate 1.

Fig. 1.

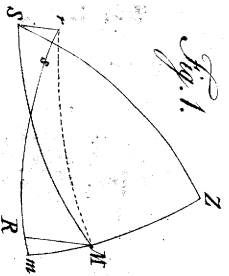


Fig. 2.

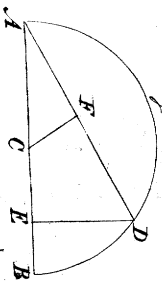


Fig. 4.

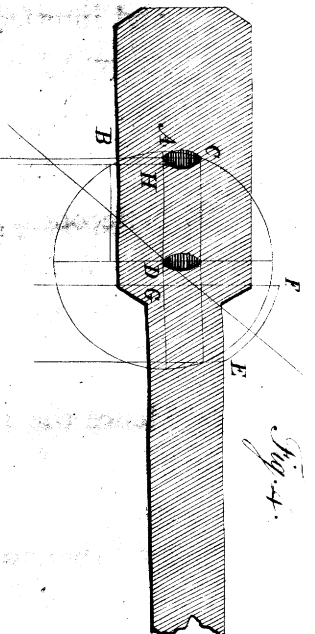


Fig. 5.

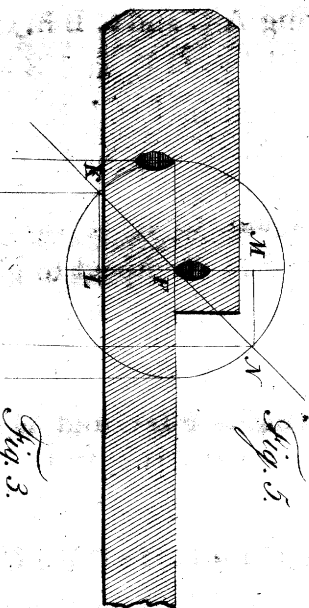


Fig. 3.

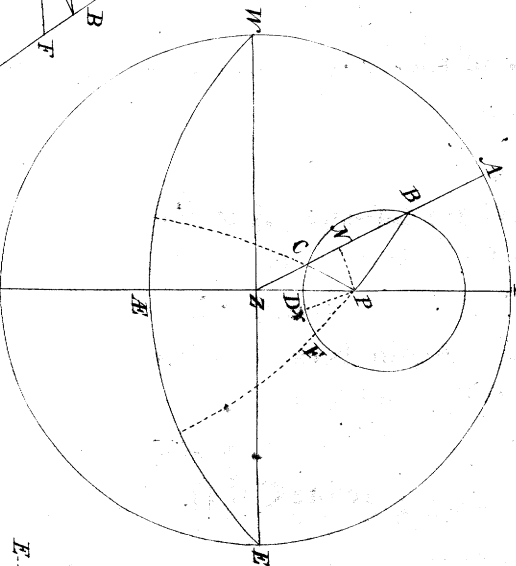


Fig. 6.

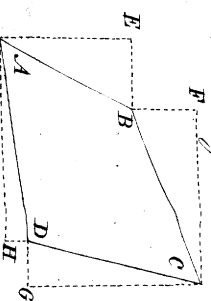


Fig. 9.

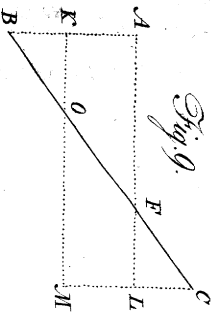


Fig. 11.

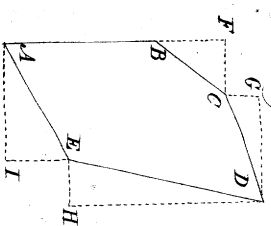


Fig. 7.

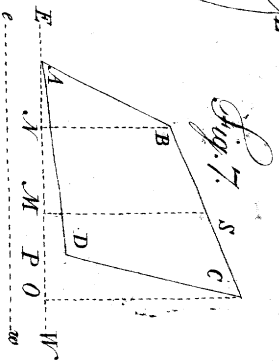


Fig. 13.



Fig. 14.

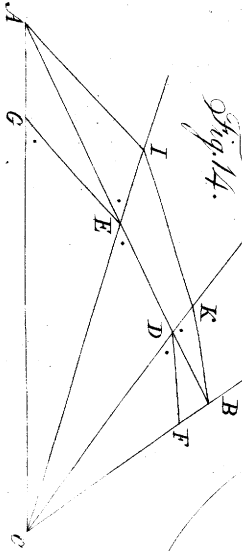


Fig. 10.

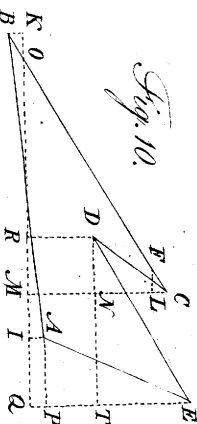


Fig. 12.

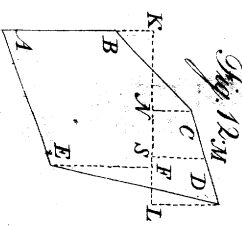
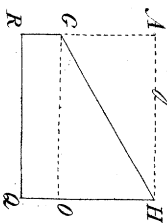


Fig. 8.



Let it be required to find two mean proportionals between any two given extremes Y and Z . (Plate 1, fig. 13th.)

Take any straight line $AC=Z$, (Plate 1, fig. 14) and from the point C draw CI making any angle ACI , less than one third of two right angles. Make the angles ICK and KCB each equal ACI , and make $CB=Y$. Join AB intersecting IC and KC in E and D . From E and D draw EG and DF , making the angles AEG and BDF each equal ACI , and intersecting AC and CB in G and F . Through A and B draw AI parallel to GE , and BK parallel DF , meeting CE and CD produced in I and K . Then are CK and CI the mean proportionals required.

Secondly, I am to shew that the demonstration, which Mr. Winthrop has attempted to give of the above method, is not true. The substance of it is as follows.

The angle $BDC=DEC+ECD$; but $BDF=ECD$. Hence the triangles FDC and DEC are similar. Also the angle $AEC=EDC+DCE$, but $AEG=DCE$: hence the triangles GEC , EDC and DFC are all similar to each other. And therefore $CF:CD:CE:CG$. Again because AI is parallel to GE , and KB parallel to DF , join IK , and IK is parallel to ED . Consequently the triangles ACI , ICK and KCB are all similar one to another; and therefore $CB(Y):CK:CI:CA(Z)$ as was required.

But although AI is parallel to GE , and KB is parallel to DF , it does not follow that IK is parallel to ED ; and therefore Mr. Winthrop's demonstration is not true.

G

I am

I am thirdly to demonstrate that Mr. Winthrop's method of performing this problem is universally false.

Because Mr. Winthrop has not proved that IK is parallel to ED , it does not follow that IK is not really parallel to ED . Imagine therefore that IK is parallel to ED . Then will the triangles $AI E$, $K B D$, $G E C$, $E D C$, $D F C$, $A I C$, $I K C$ and $K B C$ be all similar to one another. Therefore $IE : KD :: CE : CD$; and also $IE : KD :: AI : BD$. But AI is to BD in the complicated ratio of AI to KB and of KB to BD , and $AI : KB :: AC : KC :: \overline{CE}^2 : \overline{CD}^2$; also $KB : BD :: CE : CD$. Therefore $AI : BD :: \overline{CE}^3 : \overline{CD}^3$. But it has just been proved that $AI : BD :: IE : KD :: CE : CD$; and hence $\overline{CE}^3 : \overline{CD}^3 :: CE : CD$, which is absurd. IK cannot therefore be parallel to ED ; and consequently this method of performing the problem is universally false. And as Mr. Winthrop's mode of duplicating the Cube depends upon the truth of this problem, it is also universally false. Q. E. D.

The same may be demonstrated by a variety of other methods. For instance by the addition of fines, tangents, &c. or by the logarithmic spiral.

I must at the same time acknowledge that Mr. Winthrop's mistake on this subject was so natural, that I at first followed him in it. A second inspection convinced me of my mistake.

HALLOWELL, in the DISTRICT of MAINE, June 2, 1798.

N. B. Soon after the paper, to which Mr. Baron's observations relate, was communicated to the Academy, it was examined by several of the members, skilled in mathematical science, who were of opinion, that the author had not demonstrated his theorem. But at the particular request of the author the committee for publication consented to its insertion in the Memoirs.